

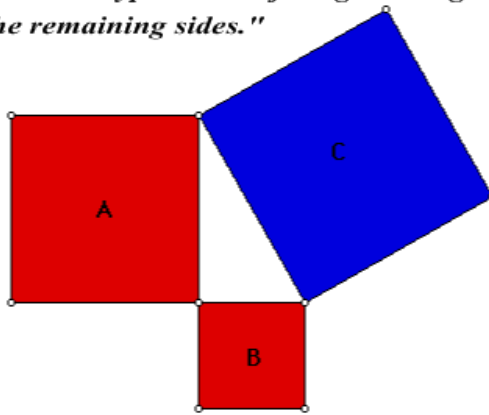
The Pythagorean Theorem is a statement about triangles containing a right angle. The Pythagorean Theorem states that:

"The area of the square built upon the hypotenuse of a right triangle is equal to the sum of the areas of the squares upon the remaining sides."

Thus, the Pythagorean Theorem stated algebraically is:

$$a^2 + b^2 = c^2$$

for a right triangle with sides of lengths a, b, and c, where c is the length of the hypotenuse.



According to the Pythagorean Theorem, the sum of the areas of the two red squares, squares A and B, is equal to the area of the blue square, square C.

if the sides of a right triangle are whole numbers, it is said to be a **Pythagorean Triple**

Generating Pythagorean Triples

There is a simple formula that gives all the Pythagorean triples.

Suppose that m and n are two positive integers, with $m < n$. Then $n^2 - m^2$, $2mn$, and $n^2 + m^2$ is a Pythagorean triple.

For example, if $n=2$ and $m=1$, we get the Pythagorean Triple 3-4-5 ($2^2 - 1^2 = 3$; $2 \times 2 \times 1 = 4$; and $2^2 + 1^2 = 5$).

It's easy to check algebraically that the sum of the squares of the first two is the same as the square of the last one. Why is it that *every* triple can be generated in this manner?

Here are the first few triples for m and n between 1 and 10. Notice any patterns?

	m= 1	2	3	4	5	6
7		8	9			

+						

n=						
2	[3, 4, 5]					
3	[8, 6, 10]	[5, 12, 13]				
4	[15, 8, 17]	[12, 16, 20]	[7, 24, 25]			
5	[24, 10, 26]	[21, 20, 29]	[16, 30, 34]	[9, 40, 41]		
6	[35, 12, 37]	[32, 24, 40]	[27, 36, 45]	[20, 48, 52]	[11, 60, 61]	
7	[48, 14, 50]	[45, 28, 53]	[40, 42, 58]	[33, 56, 65]	[24, 70, 74]	[13, 84, 85]
8	[63, 16, 65]	[60, 32, 68]	[55, 48, 73]	[48, 64, 80]	[39, 80, 89]	[28, 96, 100]
	[15, 112, 113]					
9	[80, 18, 82]	[77, 36, 85]	[72, 54, 90]	[65, 72, 97]	[56, 90, 106]	[45, 108, 117]
	[32, 126, 130]	[17, 144, 145]				
10	[99, 20, 101]	[96, 40, 104]	[91, 60, 109]	[84, 80, 116]	[75, 100, 125]	[64, 120, 136]
	[51, 140, 149]	[36, 160, 164]	[19, 180, 181]			

the Pythagorean triples highlighted below are primitive in that they cannot be reduced to a proportional ratio; for example, a 6-8-10 triple can be reduced to 3-4-5 by dividing each side by 2, so we call a 6-8-10 an imprimitive; but a 3-4-5 is in its simplest form.

There are 16 primitive triples with a hypotenuse under 100.

CAN YOU FIND THE PATTERNS IN THE PYTHAGOREAN TRIPLES OF:

3-4-5

5-12-13

7-20-21

9-40-41

11-60-61

- take the square of any odd whole number greater than 1;
- the sum of the largest leg and the hypotenuse (consecutive numbers) will be the square of the odd whole number.

8-15-17

12-35-37

16-63-65

20-99-101

24-143-145

- multiples of 4 greater than 4 may be the smallest leg of a triple.
- the sum of the larger leg and the hypotenuse which is 2 greater than the largest leg is half of the square of the shorter leg.

